# Trigonometric Inequalities (3.10) Homework

Attempt this homework by hand, but you may use a calculator/Desmos if you get stuck. For questions 8, 9, and 10, you should use a calculator/Desmos.

1. Let $f(\theta) = 3\sin\theta + 1$ for $0 \le \theta \le 2\pi$. What are all values of $\theta$, $0 \le \theta \le 2\pi$, where $f(\theta)$ is negative?

2. Let $g(x) = 2\cos^2 x - \cos x - 1$ for $-\pi \le x \le \pi$. What are all values of $x$ where $f(x)$ is positive?

3. What are all values of $\theta$, $-\pi \le \theta \le \pi$, for which $\cos\theta > -\frac{1}{2}$ and $\sin\theta < \frac{\sqrt{3}}{2}$?

4. What are all values of $\theta$, $0 \le \theta \le 2\pi$, for which $2\sin\theta < \sqrt{3}$ and $2\cos\theta > \sqrt{2}$?

Solve each inequality over the interval $0 \le \theta \le 2\pi$.

5. $10\sin\theta - 1 \le 2\sin\theta + 5$

6. $2\sin\theta\cos\theta - 3\cos\theta > 0$

7. $3\sin\theta\cos\theta - 4\sin\theta > 0$

Solve the following by analyzing a graph using Desmos or any other graphing utility.

8. The height of the tide on Fort Lauderdale Beach today can be modelled by the function $h(t) = 5\sin\left(\frac{\pi}{6}(t-7)\right) + 13$, where $t$ is in hours since midnight and $h(t)$ is in feet. Over the course of today ($0 \le t \le 24$), for approximately how many hours is the tidal height greater than $16$ feet? Round to the nearest whole hour.

9. Patty is bungee jumping! She jumps off a 200m tall building. As she falls, the bungee cord is stretched slowing her down until she reaches a height of $80$m off the ground. At that point, the cord pulls her back up to her original height. For values of $t$ in the interval $0 \le t \le 10$, Patty's height above the ground can be modelled by the function $h(t) = 60\sin\left(\frac{\pi}{5}(t-7.5)\right) + 140$ where $h(t)$ is in feet and $t$ is in seconds. For approximately how many seconds is Patty less than $120$ feet off the ground?

10. Let $q$ be the function given by $q(x) = 4\arctan(3x) + \pi$. Find all input values in the domain of $q$ that yield a positive output value.

Review:

11. Let $p(x) = 6\arctan(4x + 1)$. Find all inputs in the domain of $p$ that yield an output of $-\frac{3\pi}{2}$.

12. Evaluate $\arcsin(1)$

13. Evaluate $\arccos\left(-\frac{1}{2}\right)$

14. Evaluate $\arctan\left(-\frac{1}{\sqrt{3}}\right)$

Solve the equation over the interval $0 \leq \theta < 2\pi$. Write your answer in terms of arcsin, arccos, or arctan.

15. $4\tan\theta + 3 = 6$

16. $4\sec\theta + 13 = 6$

17. $5\sin\theta + 2\csc\theta = 7$

18. $3\sin^2\theta + 5\cos\theta = 5$

# Trigonometric Inequalities (3.10) HW Answers

Wondering whether $(0.2,\ 3.142)$ would be counted as correct if the answer is $(0.2, \pi)$? The answer is yes. As long as the first three decimal places of your answer and my answer agree, your answer is correct.

1. $\left(\pi + \arcsin\left(\frac{1}{3}\right), 2\pi - \arcsin\left(\frac{1}{3}\right)\right)$
   As decimal values, this is $(3.481, 5.943)$
2. $\left[-\pi, -\frac{2\pi}{3}\right) \cup \left(\frac{2\pi}{3}, \pi\right]$
3. $\left(-\frac{2\pi}{3}, \frac{\pi}{3}\right)$
4. $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right]$
5. $\left[0, \arcsin\frac{3}{4}\right] \cup \left[\pi - \arcsin\frac{3}{4}, 2\pi\right]$
   As decimal values, this is $[0, 0.848] \cup [2.294, 2\pi]$
6. $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$
7. $(\pi, 2\pi)$
8. The tide is greater than $16$ feet over the interval $8.229 < t < 11.771$ (about 3.5 hours)
   The tide is also greater than $16$ feet over the interval $20.229 < t < 23.771$ (another 3.5 hours)
   **There are approximately $7$ hours during which the tide is over $16$ feet.**
9. Patty is less than $120$ feet off the ground over the interval $3.041 < t < 6.959$
   **Out of the first ten seconds after Patty begins bungee jumping, she is less than $120$ feet off the ground for approximately $4$ seconds.**
10. $\left(-\frac{1}{3}, \infty\right)$
11. $x = -\frac{1}{2}$
12. $\frac{\pi}{2}$
13. $\frac{2\pi}{3}$
14. $-\frac{\pi}{6}$
15. $\theta = \arctan\left(\frac{3}{4}\right)$ or $\theta = \pi + \arctan\left(\frac{3}{4}\right)$
16. $\theta = \pi - \arccos\left(\frac{4}{7}\right)$ or $\theta = \pi + \arccos\left(\frac{4}{7}\right)$
17. $\theta = \arcsin\left(\frac{2}{5}\right)$ or $\theta = \frac{\pi}{2}$ or $\theta = \pi - \arcsin\left(\frac{2}{5}\right)$
18. $\theta = 0$ or $\theta = \arccos\left(\frac{2}{3}\right)$ or $\theta = 2\pi - \arccos\left(\frac{2}{3}\right)$